On the total weight choosability of graphs *†

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Abstract

A graph $G = (V, E)$ is called $(k, \{1, \ldots, k'\})$-*total weight choosable* if for every total list assignment L which assigns to each vertex v a set $L(v)$ of k real numbers, and assigns to each edge e the set $L(e) = \{1, \ldots, k'\}$, there is a mapping $f : V \cup E \to \mathbb{R}$ such that $f(y) \in L(y)$, for every $y \in V \cup E$ and for every two adjacent vertices x and x', $\sum_{u\in N(x)} f(xu) + f(x) \neq \sum_{u\in N(x')} f(x'u) + f(x')$. In 2004, it was proved that every 3-colorable graph is 3-weight colorable. We extend this result by showing that every 3-colorable graph is $(1, \{1, 2, 3\})$-total weight choosable. Also, we prove that every bipartite graph is $(2, \{1, 2\})$-total weight choosable.

1 Introduction

Throughout this paper, all graphs are simple with no loops and multiple edges. Let G be a graph. We denote the vertex set and the edge set of G by V and E, respectively. For every $v \in V$, $N(v)$ denotes the set of neighbors of v and define $d(v) = |N(v)|$. The complete graph of order n and the path of order n are denoted by K_n and P_n, respectively. A $u - v$ *walk* is defined as a sequence of vertices starting at u and ending at v, where the consecutive vertices in the sequence are adjacent.

A vertex coloring is an assignment of colors to the vertices of a graph such that no two adjacent vertices have the same color. Let G be a graph, $S \subseteq \mathbb{R}$ and $\alpha : E \to S$ be a weighting of the edges of G. The graph G is called *weight colorable*, if the weight degrees $s(v) = \sum_{u\in N(v)} \alpha(uv)$ of the vertices yield a proper vertex coloring of G. Let k be a natural number. The graph G is said to be k-*weight colorable*, if it is weight colorable for $S = \{1, \ldots, k\}$. The following conjecture was proposed in [5].

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Conjecture 1. *Every connected graph $G \neq K_2$ is 3-weight colorable.*

In [5], the authors proved that every connected graph, except K_2, is weight colorable for any set $S \subseteq \mathbb{R}$ of size at least 183, provided S is independent over the field of rational numbers. With this restriction, the bound was reduced to 4 by Addario-Berry et al. [1]. Notice that this does not imply that there is a finite set of integers doing the same job. However in [2], it was proved that $\{1, \ldots, 30\}$ is enough. Recently, it has been proved that the aforementioned conjecture is true for $\{1, 2, 3, 4, 5\}$, see [4].

A graph G is called *k-edge-weight-choosable* if for any *list assignment* L which assigns to each edge e a set $L(e)$ of k real numbers, G has a proper edge weighting f such that $f(e) \in L(e)$, for each $e \in E$. In [3], Bartnicki, Grytczuk and Niwczyk conjectured that every connected graph $G \neq K_2$ is 3-edge-weight-choosable and showed that the conjecture is true for the complete graphs and the complete bipartite graphs.

Consider a graph $G = (V, E)$. A mapping $f : V \cup E \to \mathbb{R}$ is called a *total weighting*. A total weighting is called *proper* if the weight degree of v, $f(v) + \sum_{u \in N(v)} f(uv)$ yields a proper vertex coloring of G. Kalkowski, Karoński and Pfender proved that every simple graph other than K_2 has a proper total weighting f such that $f(v) \in \{1, 2\}$, for every $v \in V$ and $f(e) \in \{1, 2, 3\}$, for every $e \in E$ in [4]. Przybyło and Woźniak proposed the following conjecture named $1, 2$-Conjecture in [7].

Conjecture 2. *Every graph $G = (V, E)$ has a proper total weighting f such that $f(y) \in \{1, 2\}$, for every $y \in V \cup E$.*

They proved that the conjecture is true for the complete graphs, 4-regular graphs and 3-colorable graphs.

A graph G is called *(k, k')-total weight choosable* if for any total list assignment L which assigns to each vertex v a set $L(v)$ of k real numbers and assigns to each edge e a set $L(e)$ of k' real numbers, there is a mapping $f : V \cup E \to \mathbb{R}$ such that $f(y) \in L(y)$, for every $y \in V \cup E$ and for any two adjacent vertices x and x', $f(x) + \sum_{u \in N(x)} f(ux) \neq f(x') + \sum_{u \in N(x')} f(ux')$. In [9], Wong and Zhu proposed the following conjecture which is stronger than $1, 2$-Conjecture.

Conjecture 3. *Every graph is $(2, 2)$-total weight choosable and every graph with no isolated edge is $(1, 3)$-total weight choosable.*

The $(2, 2)$-total weight choosability of the complete graphs, trees, and generalized theta graphs are proved in [9]. It is also proved that the complete graphs, the complete bipartite graphs and trees other than K_2 are $(1, 3)$-total weight choosable, see [3]. Later in [8], they proved the $(2, 2)$-total weight choosability of complete multipartite graphs of the form $K_{n,m,1,1,\ldots,1}$ and $(1, 2)$-total weight choosability of the complete bipartite graphs other than K_2.

Let $a_1, \ldots, a_{k'} \in \mathbb{R}$. A *$(k, \{a_1, \ldots, a_{k'}\})$-total list assignment* L is an assignment of a list of k real numbers $L(v)$ to each vertex v, and the list $L(e) =$

$\{a_1, \ldots, a_{k'}\}$ to each edge e. Consider a $(k, \{a_1, \ldots, a_{k'}\})$-total list assignment L. A $(k, \{a_1, \ldots, a_{k'}\})$-total weighting f is a mapping $f : V \cup E \to \mathbb{R}$, such that $f(y) \in L(y)$ for any $y \in V \cup E$.

A graph $G = (V, E)$ is called $(k, \{a_1, \ldots, a_{k'}\})$*-total weight choosable* if for any $(k, \{a_1, \ldots, a_{k'}\})$-total list assignment L, there is a proper $(k, \{a_1, \ldots, a_{k'}\})$-total weighting f; i.e. for any two adjacent vertices x and x', $\sum_{u \in N(x)} f(xu) + f(x) \neq \sum_{u \in N(x')} f(x'u) + f(x')$.

A k*-edge-weighting* is an assignment of an integer weight from $\{1, \ldots, k\}$ to each edge. Recently, in [6], Lu. et al. showed that for a given 3-connected bipartite graph and a vertex-coloring c_0 by the abelian group $\mathbb{Z}_2$, there exists a 2-edge-weighting by $\mathbb{Z}_2$, such that the induced vertex coloring is c_0. We show that if a k-colorable graph G admits a vertex-coloring k-edge-weighting by the abelian group $\mathbb{Z}_k = \{1, \ldots, k\}$, then G is $(1, \{1, \ldots, k\})$-total weight choosable and so every 3-connected bipartite graph is $(1, \{1, 2\})$-total weight choosable.

In this paper, we prove that every 3-colorable graph is $(1, \{1, 2, 3\})$-total weight choosable and every bipartite graph is $(2, \{1, 2\})$-total weight choosable.

Consider a graph $G = (V, E)$. Throughout this paper, $f : V \cup E \to \mathbb{R}$ is a function, where $f(v)$ denotes the weight of vertex v chosen from $L(v)$, for every $v \in V$ and $f(e)$ denotes the weight of edge e chosen from $L(e)$, for every $e \in E$. For every $v \in V$, we define $w(v) = f(v) + \sum_{u \in N(v)} f(uv)$ which denotes the *weight color* of the vertex v.

2 Three preliminary lemmas

In this section, we prove some lemmas which will be used in the proof of our theorems. The first lemma shows that if a graph G is total weight choosable by every integer total list assignment, then it is total weight choosable for every total list assignment which assigns list of real numbers to the vertices, and assigns list of integers to the edges.

Lemma 1. *Let $G = (V, E)$ be a graph. If G is (k, k')-total weight choosable for every integer total list assignment L', then G is (k, k')-total weight choosable for every total list assignment L which assigns to each vertex v a list $L(v) \subseteq \mathbb{R}$ and to each edge e a list $L(e) \subseteq \mathbb{Z}$.*

Proof. Let $T = \cup_{x \in V} L(x)$. If $|T| = 1$, then let $\varepsilon = 1$, otherwise, let $\varepsilon = \min_{a \neq b \in T} |a - b|$. Let $\alpha \geq \lceil \frac{1}{\varepsilon} \rceil$ and define $L'(x) = \{\lfloor \alpha y \rfloor \,|\, y \in L(x)\}$, for every $x \in V$ and $L'(e) = \{\alpha y \,|\, y \in L(e)\}$, for every $e \in E$. It is clear that L' is an integer (k, k')-total list assignment.

By assumption, there is a proper total weighting f' for L'. Suppose that f is a total weighting for L such that $\lfloor \alpha f(x) \rfloor = f'(x)$, for each $x \in V$ and $f(e) = \alpha^{-1} f'(e)$, for each $e \in E$. Obviously, $f(x)$ is uniquely defined for every $x \in V$.

Toward a contradiction, assume that there are two adjacent vertices u and v with the same weight colors. Thus we have:

$$f(u) + \sum_{x \in N(u)} f(ux) = f(v) + \sum_{x \in N(v)} f(vx).$$

This implies that

$$\lfloor \alpha f(u) + \sum_{x \in N(u)} \alpha f(ux) \rfloor = \lfloor \alpha f(v) + \sum_{x \in N(v)} \alpha f(vx) \rfloor.$$

Since $\alpha f(e) = f'(e) \in \mathbb{Z}$, for every $e \in E$, we have:

$$\lfloor \alpha f(u) \rfloor + \sum_{x \in N(u)} f'(ux) = \lfloor \alpha f(v) \rfloor + \sum_{x \in N(v)} f'(vx),$$

and so we find

$$f'(u) + \sum_{x \in N(u)} f'(ux) = f'(v) + \sum_{x \in N(v)} f'(vx),$$

a contradiction. This implies that f is a proper total weighting for L. □

With the same proof of the Lemma 1, we have the following remark.

Remark 2. *Let $G = (V, E)$ be a graph, $\{a_1, \ldots, a_{k'}\} \subseteq \mathbb{Z}$ amd k is a positive integer. If G is $(k, \{\alpha a_1, \ldots, \alpha a_{k'}\})$-total weight choosable for every integer total list assignment and any odd integer α, then G is $(k, \{a_1, \ldots, a_{k'}\})$-total weight choosable.*

Consider a 3-connected bipartite graph G and an arbitrary vertex coloring c_0 (which is not necessarily proper) in the abelian group $\mathbb{Z}_2 = \{0, 1\}$. In [6], it is shown that there is a 2-edge weighting α in $\mathbb{Z}_2$, such that the induced vertex coloring (i.e. $\sum_{x \in N(v)} \alpha(xv)$ for each vertex v) is c_0. Let k be a natural number and G be a k-colorable graph. In the following lemma, we show that if for every vertex coloring c_0, there is a k-edge weighting such that the induced vertex coloring is c_0, then G is $(1, \{1, \ldots, k\})$-total weight choosable.

Lemma 3. *Let G be a k-colorable graph and let c_0 be an arbitrary vertex coloring in the abelian group $\mathbb{Z}_k = \{1, \ldots, k\}$. If there is a k-edge weighting in $\mathbb{Z}_k$ such that the induced vertex coloring is c_0, then G is $(1, \{1, \ldots, k\})$-total weight choosable.*

Proof. Consider a $(1, \{1, \ldots, k\})$-total list assignment L for G. Clearly, every total weighting f has this property $L(x) = \{f(x)\}$, for every $x \in V$. By Lemma 1, we may assume that L is an integer list assignment. Let $c : V \to \mathbb{Z}_k$ be a proper vertex coloring of G and $c_0 : V \to \mathbb{Z}_k$ be another vertex coloring such that for every $x \in V$, $c_0(x) = c(x) - (f(x) \mod k)$. By assumption there exists a k-edge weighting $\Phi : E \to \mathbb{Z}_k$ such that the induced vertex coloring is c_0. Now, let $f(e) = \Phi(e)$, for every $e \in E$. It is obvious that $w(x) \equiv c(x)\,(mod\ k)$, for every $x \in V$ and we are done. □

The following lemma is about the total weight choosability of bipartite graphs under some conditions. Suppose that $G = (X, Y)$ is a connected bipartite graph and we have an integer total list assignment. Let $H = \{v \in X \mid w(v) \equiv 0\,(mod\ 2)\} \cup \{v \in Y \mid w(v) \equiv 1\,(mod\ 2)\}$.

Lemma 4. *Let G be a connected graph. Consider an integer $(1,2)$-total list assignment of G such that for each edge two integers in its list have different parities. Then there exists a total weighting of G such that $|H| \in \{0, 1\}$.*

Proof. For each $e_i \in E$, let $L(e_i) = \{r_i, s_i\}$, where r_i is odd and s_i is even. First we assign the weight r_i to every edge of G. If $|H| \geq 2$, then choose two vertices u and v from H and consider a path $u = v_1, \ldots, v_k = v$ in G. In this path, change the weight of each edge to r_i if its weight is s_i and conversely. Note that, the parities of $w(u)$ and $w(v)$ will be changed whereas the parity of $w(v_i)$, $2 \leq i \leq k-1$ is preserved. Now, update H. If in the start of algorithm $|H|$ is even, then after finitely many steps $H = \emptyset$ and $w(x)$ is odd, for every $x \in X$ and $w(y)$ is even, for every $y \in Y$. Therefore we have a proper total weighting of G. If in the start of algorithm $|H|$ is odd, then in the last step $|H| = 1$. □

Remark 5. *Let $H = \{v\}$ and $u \in V$ be an arbitrary vertex. Consider a path between u and v in G. Change the weight of each edge of this path from r_i to s_i and conversely. So for each $u \in V$, one may assume that $H = \{u\}$. We call this vertex a bad vertex.*

3 $(1, \{1, 2, 3\})$-total weight choosability of 3-colorable graphs

In [5], it was proved that every 3-colorable graph is 3-weight colorable. In this section, we extend this result by showing that every 3-colorable graph is $(1, \{1, 2, 3\})$-total weight choosable.

Theorem 6. *Every connected 3-colorable graph except K_2 is $(1, \{1, 2, 3\})$-total weight choosable.*

Proof. Let G be a 3-colorable graph and $V = \{v_1, \ldots, v_n\}$ and $L(v_i) = \{t_i\}$. Define $L'(v_i) = \{\lfloor t_i \rfloor\}$ for every $v_i \in V$ and $L'(e) = L(e)$ for each $e \in E$. It suffices to show that G is $(\{\lfloor t_i \rfloor\}, \{1, 2, 3\})$-total weight choosable, because if there is a total weighting for L' with weight colors w' and u and v are two adjacent vertices, then $|w'(u) - w'(v)| \geq 1$. Now, replace each $\lfloor t_i \rfloor$ with t_i and call the new weight color by w. Since $0 \leq t_i - \lfloor t_i \rfloor < 1$, $1 \leq i \leq n$, $w(u) \neq w(v)$.

First, we prove the theorem for 3-partite graphs which are not bipartite. Consider the group $\mathbb{Z}_3 = \{1, 2, 3\}$. If one can find a proper total weighting of G by $\mathbb{Z}_3$, then it is a proper total weighting of G by the list $\{1, 2, 3\}$, for every edge. Let $c : V \to \mathbb{Z}_3$ be a proper vertex coloring and f be a total weighting in $\mathbb{Z}_3$. For every $x \in V$, define $g_x = c(x) - f(x)$. Since $g_x \in \mathbb{Z}_3$, there exists $h \in \mathbb{Z}_3$ such that $\sum_{x\in V} g_x = 2h$. To every edge of G assign the weight 3 except one edge and consider the weight h for this edge. This implies that $\sum_{x\in V}(w(x)-f(x)) = 2h$.

We claim that there exists a weighting to the edges of G such that for every $x \in V$, $w(x) = c(x)$. Suppose that there are two vertices u and v such that $c(u) \neq w(u)$ and $c(v) \neq w(v)$. Since G is not bipartite, there exists an $u-v$ walk of even length, say, $p_1, \ldots, p_k$. For every edge p_ip_{i+1} of this walk if i is odd, then replace $f(p_ip_{i+1})$ with $f(p_ip_{i+1}) - w(u) + c(u)$. If i is even, then replace $f(p_ip_{i+1})$ with $f(p_ip_{i+1}) + w(u) - c(u)$. Therefore, $w(u) = c(u)$ and $\sum_{x\in V}(w(x) - f(x)) = 2h$ does not change. Thus, the weight of every vertex except u and v does not change. By repeating this procedure, ultimately we have at most one vertex $v \in V$ such that $w(v) \neq c(v)$. If there exists exactly one such a vertex, then the following holds.

$$\sum_{x\in V} g_x = 2h = \sum_{x\in V}(w(x) - f(x)).$$

For every $x \neq v$, $w(x) = c(x)$. This yields that

$$\sum_{x\in V}(c(x) - f(x)) = \sum_{x\in V}(c(x) - f(x)) + w(v) - c(v),$$

and so $w(v) - c(v) = 0$, a contradiction.

Now, let $G = (X, Y)$ be a bipartite graph. First, we assign the weight 1 to every edge of G.

According to the Lemma 4, we can set the weight of edges such that $|H| = 0$ or $|H| = 1$. If $|H| = 0$, then there is nothing to prove. Thus assume that $H = \{u\}$. Already, for every $e \in E$, $f(e) \in \{1, 2\}$. The only bad case is $w(u) = w(u')$, for some $u' \in N(u)$. Now, define $m = \max_{x\in V}(2d(x) + f(x))$ and $M = \{\, x \in V \mid 2d(x) + f(x) = m \,\}$. Suppose that there exists a vertex $v \in M$ such that at least one of the following cases occurs:

1) $N(v) \cap M = \emptyset$.

2) There exists an edge vz with weight 1 such that $z \notin M$.

3) There are at least two edges incident with v whose weights are 1.

Now, by remark, one may assume that v is bad. We change the weight of each edge incident with v to 3 if it is 1.

Thus, already $w(v)$ is at least one more than the weight of its neighbors, and the parity of the weight of each vertex does not change and so we are done.

Now, assume that none of the Cases $1, 2, 3$ occur. Consider the following cases:

4) There exists a vertex $v_1 \in M$ such that v_1 lies on a cycle. We can assume that v_1 is a bad vertex. Now, we consider two cases:

(i) If there exists a cycle containing two edges of the cycle, say v_1p and v_1q, such that $f(v_1p) = f(v_1q) = 2$, then by changing 1 to 2 and conversely for the edges of this cycle the parity of each vertex does not change and $f(v_1p) = f(v_1q) = 1$ which we have argued in the Case 3 before.

(ii) If the Case (i) does not occur, then for every two edges v_1p and v_1q in every cycle their weights are 1 and 2, respectively. Now, with no loss of generality, assume that $f(v_1p) = 1$. Let $v_2 = p$. If $v_2 \notin M$, then the Case 2 holds. Thus assume that $v_2 \in M$. By changing $f(v_1v_2)$ to 2, v_2 becomes a bad vertex. Repeat this procedure. If there is just an edge with weight 1 adjacent to v_i, say v_iv_{i+1}, then we have $v_{i+1} \in M$ and every cycle containing v_i contains the edge v_iv_{i+1}(otherwise (i) holds for vertex v_i).

Change $f(v_iv_{i+1})$ to 2 as shown in Figure 1 Part(a). Then the bad vertex becomes v_{i+1}. By continuing this procedure, we do not find a repeated vertex. Because in the transformation of every bad vertex from each vertex v_i, the weight of each edge incident with v_i is 2 and before the transformation of bad vertex to v_{i+1}, there exists an edge incident with v_i whose weight is 1. Finally, we find a vertex that satisfies at least one of the Cases $1, 2, 3$ and (i), as desired.

5) The induced subgraph on M is a forest and each vertex of M is a cut vertex of G. Now, two cases may be considered:

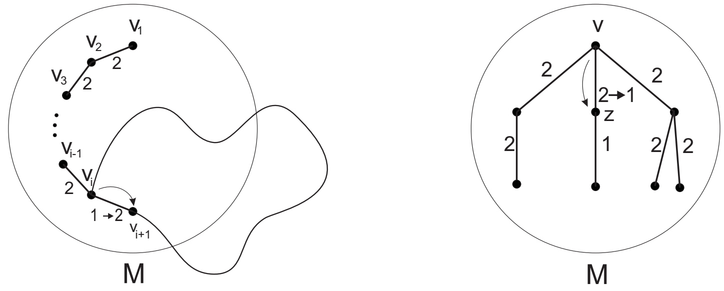
(i) There exists a vertex $v \in M$ such that $|N(v) \cap M| \geq 2$. We assume that v is a bad vertex.

If one of the neighbors of v in M, say z, has an incident edge except vz with weight 1 and $f(vz) = 2$, then we transfer the bad vertex to z by changing $f(vz)$ to 1. Hence the Case 3 occurs for z (See Figure 1 Part(b)). Otherwise, change the weight of each edge incident with v, say uv, $u \in M$ to 3. Therefore $w(v) \geq m+2$ and the weight of each vertex in $N(v) \cap M$ is $m+1$ or parity does not change. Since $N(v)$ is an independent set we are done.

(ii) Assume that for every x, $x \in M$, $|N(x) \cap M| = 1$. Thus, the induced subgraph on M is a matching. Since $G \neq K_2$, thus there exists a vertex $v \in M$ such that $d(v) \geq 2$. Assume that $u \in M$ and $z \notin M$ are two neighbors of v. Since v is a cut vertex, by removing v from V there is a connected component containing z, say S. Now, three cases can be considered:

(a) If S is just the isolated vertex z, then transfer the bad vertex to v and then let $f(vz) = 3$. If $f(uv) = 1$, then change the weight of uv to 3. Otherwise, do not change its weight. Thus $w(v) \geq m+1$, $w(z) \leq m$ and $w(u) < w(v)$.

(b) If S contains exactly two vertices z and t and $tz \in E$, then if $f(t) \neq f(z)+3$, change the weight of vz to 3. If $f(uv) = 1$, then change its weight to 3; otherwise, do not change its weight. Now, we have $w(v) \geq m+1$, $w(v) > w(u)$ and $w(z) \leq m$. Since $f(t) \neq f(z)+3$, we conclude that $w(z) \neq w(t)$ and we are done. If $f(t) = f(z)+3$, then transfer the bad vertex to t and so $f(zt) \neq 3$,



(a) (b)

Figure 1

$w(t) \neq w(z)$.

(c) Now, assume that S contains at least three vertices. Now, apply the induction on $n = |V|$. For P_3 the assertion is trivial. Suppose that the assertion is true for every graph with less than n vertices. Suppose that L'' is a $(1, \{1, 2, 3\})$-total list assignment of S. Change the weights of uv and vz to 3 and for every $s \in S \backslash \{z\}$, define $L''(s) = L'(s)$ and $L''(z) = \{f(z) + 3\}$. Now, apply the induction hypothesis for the graph S with total list assignment L''. Since vz is a cut edge, the only bad case is that $w(v) = w(z)$. In this case, change the weight of uv to 2. Therefore $w(v) \neq w(z)$. But we have $w(v) \geq m + 1$, $w(u) \geq m$ and $w(v) > w(u)$. So the weight of no vertex is equal to the weight of u or v. The proof is complete. □

4 $(2, \{1, 2\})$-total weight choosability of bipartite graphs

In [7], it has been proved that if one assigns the list $\{1, 2\}$ to each edge and each vertex of a 3-colorable graph, a complete graph or a 4-regular graph, then it has a proper total weighting. In this section we prove that every bipartite graph is $(2, \{1, 2\})$-total weight choosable.

Theorem 7. *Every bipartite graph G is $(2, \{1, 2\})$-total weight choosable.*

Proof. Let L be a $(2, \{1, 2\})$-total list assignment with $L(x) = \{a_x, b_x\}$, for each vertex x, where $a_x < b_x$. By Remark 2, it suffices to prove that G has a proper $(2, \{\alpha, 2\alpha\})$-total weighting for every integers a_x, b_x and odd integer α. Suppose that f is a $(2, \{\alpha, 2\alpha\})$-total weighting of G. First, assign the weight a_x to each vertex x i.e. $f(x) = a_x$. Since G is bipartite and parity of every pair in each list is different, according to Lemma 4 the weights of edges can be chosen such that there is at most one bad vertex. If there is no bad vertex, then we are done. Otherwise assume that there is exactly one bad vertex. If for one vertex such as u, the parities of a_u and b_u are different, then move the bad vertex to u and change $f(u)$ from a_u to b_u. So in this case we are done.

Now, assume that for every $x \in V$, $a_x \equiv b_x \pmod 2$. Assume that u is a bad vertex and $S = \{x \in N(u) | w(x) = w(u)\}$. For each $x \in S$, change $f(x)$ from

a_x to b_x. Since $a_x \equiv b_x \pmod 2$, we conclude that the parity of each element of S does not change, and $a_x \neq b_x$, then $w(u) \neq w(z)$, for each $z \in N(u)$. This completes the proof. □

Remark 8. *By the same method one can show that every bipartite graph G is $(2, \{r, s\})$-total weight choosable for every integers r and s with different parities.*

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